An Overview of Standard Credit Metrics

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Outline

- Introduction

- Monte Carlo

- Counterparty Credit Metrics
  - Exposure
  - Credit Losses

- Summary

- Glossary, endnotes, and references
Introduction: What is Counterparty Credit Risk?

“Counterparty [credit] risk is the risk that the counterparty to a trade or trades could default before the final settlement of the transaction’s cashflows” (1)

“…bilateral credit risk of transactions with uncertain exposures that can vary over time with the movement of underlying market factors” (2)

“Exposure” is the replacement cost of the trade less any credit offset. If no recovery, it is the maximum of the mark-to-market or zero

Lots of information available in the banking literature
Introduction: What is Counterparty Credit Risk?

- **Capital Adequacy**
  - Identify collateral and margining requirements, including the effects of triggers that are a function of credit rating or the price of a credit default swap
  - Estimate the cost of funding hedges and the likelihood of a liquidity squeeze

- **Risk Management**
  - Know and understand the counterparty exposure
  - Proactively identify signs of distress in individual firms or industries
  - Determine appropriate credit charge for each class of counterparty
  - “Blow-up” risk: identify scenarios that would cause the franchise to be cancelled

- **Compliance**
A trader sells fixed price gas 12 months forward to a customer, and hedges by buying fixed price gas 12 months forward at a $0.10 discount from someone else.

Is this a risky trade?
Introduction: Offsetting Trade

Source: Nymex settle
Introduction: Counterparty Credit Exposure

Market Risk View

Credit Exposure View

Hedge Cum. Exposure

Hedge Daily PnL Change

Trade Daily PnL Change

Trade Cum. Exposure
Introduction: Gasoline Crack

Source: Nymex settle
Introduction: Counterparty Credit Exposure

- Exposure concepts are intuitive
  - Simplest definition is max(MTM,0)

- The metrics suite is designed to measure different aspects of exposure, both now and in the future

- Most metrics are based on Monte Carlo
What is Monte Carlo?

- Monte Carlo is a numerical process based upon repeated random sampling to obtain a numerical result.

- The samples represent realizations of random variables, and are used to either solve mathematical problems or create possible outcomes.
  - The random variable correspond to all sorts of things, such as stock prices returns, rainfall, traffic accidents, etc.

- Surprisingly useful.
Monte Carlo: Examples

- Estimation of $\pi$
- Option pricing
- Value at risk
Monte Carlo: Estimation of $\pi$

Consider a circle that is inscribed in a square

- Area of a circle: $\pi r^2$

- Area of a square: length $\times$ width or $(2\times r) \times (2\times r)$ or $4r^2$

- Ratio of circle to square: $\pi/4$
Monte Carlo: Estimation of π
Monte Carlo: Estimation of π

What if we randomly choose points on the chart (i.e., sample) and count the number of times that the point lies in the circle?

- “x” and “y” coordinate is uniform[-1,1]

The probability of a point landing in the circle can be found two ways

- Ratio of the area of the circle to the area of the square
- Ratio of the number of times a sample lies in the circle to the total number of samples (P)

\[ \pi = 4 \times P \]
Monte Carlo: Estimation of $\pi$
Monte Carlo: Estimation of $\pi$

- 20,000 samples produces an estimate of 3.135

- Generating 20,000 fresh samples produces slightly different estimates
  - 3.1278
  - 3.1512
  - 3.1294
  - 3.1408

- The variation in the estimate is known as sampling error, and is a natural consequence of monte carlo
Generating Price Scenarios

- We can use a similar idea to generate price scenarios, but need to incorporate time as well
  - Stated differently, we need to generate samples that represent prices or price returns over time

- This is most easily done using a stochastic process

- In non-mathematical terms, a stochastic process can be thought of as an equation that generates price scenarios
  - Reasonable scenarios create reasonable results
Generating Price Scenarios: Standard Model

\[ dS = \mu S dt + \sigma S dB(t) \]

\[ \frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \]

Analytical Trees Numerical

Monte Carlo

\[ S_{t+1} = S_o e^{\left( r - \frac{1}{2} \sigma^2 \right) t + \sigma \sqrt{t} dz} \]

\[ S_{t+1} = S_o e^{\sigma \sqrt{t} dz} \]

\[ VaR = -1.65 \ast S_0 \sigma \sqrt{t} = -1.65 \sigma_{std} \]

Many different stochastic processes available
Generating Price Scenarios

\[ S_{t+1} = S_o e^{\left( r - \frac{1}{2} \sigma^2 \right) t + \sigma \sqrt{t} dz} \]

- **Starting Price**
- **Simulated price**
- **Risk free interest rate**
- **Volatility**
- **Time (forecast horizon)**
- **Gaussian random variable**

2.71828…
Gaussian (Normal) Random Variable

“dz” represents a sample from a Gaussian distribution

In Excel, it is approximated by
= NORMSINV(RAND())

It is this term that creates the randomness in the formula
Gaussian Random Variable

Frequency
(10,000 samples)

Counts

Probability Density

Frequency Count

Counts

0 50 100 150 200 250 300 350 400 450

0.45 0.4 0.35 0.3 0.25 0.2 0.15 0.1 0.05 0 0

-4 -3 -2 -1 0 1 2 3 4

-4 -3 -2 -1 0 1 2 3 4
Generating Price Scenarios

Day 0  Day 1  Day 2  Day 3  Day 4  Day 5  Day 6  Day 7  Day 8  Day 9  Day 10
Monte Carlo: Option Pricing

Monte Carlo provides a simple way to value European options

Recipe

- Generate price scenarios for the option’s time to maturity
- For each scenario, calculate the option payoff. For a call option, this equals \( \max(S-K, 0) \)
- Average the option payoff over all scenarios, and discount to today
Monte Carlo: Option Pricing

<table>
<thead>
<tr>
<th>Starting Price</th>
<th>$50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strike</td>
<td>$50</td>
</tr>
<tr>
<td>Risk free rate</td>
<td>1%</td>
</tr>
<tr>
<td>Volatility</td>
<td>15%</td>
</tr>
<tr>
<td>Time to Maturity</td>
<td>0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scenario</th>
<th>dz</th>
<th>Simulated Price</th>
<th>Call Option Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.533</td>
<td>$44.89</td>
<td>$0.00</td>
</tr>
<tr>
<td>2</td>
<td>-1.284</td>
<td>$45.74</td>
<td>$0.00</td>
</tr>
<tr>
<td>3</td>
<td>0.2139</td>
<td>$51.18</td>
<td>$1.18</td>
</tr>
<tr>
<td>4</td>
<td>-0.303</td>
<td>$49.23</td>
<td>$0.00</td>
</tr>
<tr>
<td>5</td>
<td>-0.72</td>
<td>$47.71</td>
<td>$0.00</td>
</tr>
<tr>
<td>6</td>
<td>1.7226</td>
<td>$57.31</td>
<td>$7.31</td>
</tr>
<tr>
<td>7</td>
<td>0.7281</td>
<td>$53.19</td>
<td>$3.19</td>
</tr>
<tr>
<td>8</td>
<td>-0.474</td>
<td>$48.60</td>
<td>$0.00</td>
</tr>
<tr>
<td>9</td>
<td>1.6841</td>
<td>$57.14</td>
<td>$7.14</td>
</tr>
<tr>
<td>10</td>
<td>0.4565</td>
<td>$52.11</td>
<td>$2.11</td>
</tr>
</tbody>
</table>
Monte Carlo: Value-at-Risk

What is VaR?
- A percentile on a loss distribution, usually the 5th percentile: “I am 95% sure that I will not lose more than $20MM over the next 24 hours”
- Assumes that positions remain fixed over the simulation horizon

VaR can be estimated using similar techniques
- Simulate prices for a much shorter time horizon (usually one day)
- Calculate changes in the mark-to-market of each trade instead of an option payoff

Recipe
- Simulate prices for the desired forecast horizon
- For each trade, calculate the change in MTM based on the simulated price
- VaR is the 5th percentile of the distribution of the change in MTM
Monte Carlo: VaR Example

Example: 100 shares of COP purchased at $50; 10 day VaR

<table>
<thead>
<tr>
<th>Scenario</th>
<th>New Price</th>
<th>Change in Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$57.82</td>
<td>$281.84</td>
</tr>
<tr>
<td>2</td>
<td>$54.35</td>
<td>-$64.58</td>
</tr>
<tr>
<td>3</td>
<td>$49.56</td>
<td>-$544.34</td>
</tr>
<tr>
<td>4</td>
<td>$58.83</td>
<td>$382.93</td>
</tr>
<tr>
<td>5</td>
<td>$49.73</td>
<td>-$526.62</td>
</tr>
</tbody>
</table>

Sort the change in value from worst loss to best gain to create the empirical distribution function
Monte Carlo: VaR Example

5% chance of losing at least $518 in the next 10 days
Counterparty Credit Metrics

- Similar techniques are used to estimate counterparty credit metrics

- Exposure metrics

- Credit loss events
  - Credit value adjustment (CVA)

- Potential collateral and margin requirements (FVA)
Exposure Metrics

- Large number of metrics are discussed in the literature
  - Current Exposure (CE)
  - Potential Future Exposure (PFE)
  - Expected Exposure (EE)
  - Expected Positive Exposure (EPE)
  - Effective Expected Positive Exposure (Effective EPE)
  - Peak Exposure

- Formal definitions given in the glossary (pg 43)
  - Key concept: exposure = max(MTM, 0)

- Two examples
  - One barrel of oil sold one month forward
  - 12 month strip sold forward
Example: Forward Sale

PFE(t) at 95% confidence. Every point is the 95th percentile of the distribution function for that particular day.

EPE (avg. of EE(t)) = $1.71
Example: 12 Month Strip

- Two competing effects
  - Price uncertainty increases over time
  - Volume amortizes as contracts roll off

- PFE(t) is max(MTM(t),0) at 95% confidence

- EE(t) is the average exposure at time t

- EPE is the average of EE(t) ~$47.56
Effective EPE is the average of Eff. EE(t)

Example: 12 Month Strip

Peak Exposure

PFE(t)

Effective EE(t)

EE(t)
Funding Liquidity Risk

Estimates for funding liquidity can be generated in a similar way, especially if collateral and margin requirements are formula-based.

Details matter: just be clear what you are after

What details?
- Margin thresholds
- Minimum transfer amounts
- Time interval between request for additional collateral and when counterparty is considered in default (margin period of risk)
Estimating Credit Losses

If the exposure metrics and default probabilities are readily available, loss distributions can be estimated with “just a bit more work”

Default probabilities are and will likely remain a hard problem

- External Credit Ratings
- Scoring Models
- Market Observables
  - Bond spreads
  - Merton model
  - Credit derivatives
- Expert judgment if no information available
Default Probabilities

Default probabilities can be gleaned from the prices of certain debt instruments (subject to certain assumptions).

Two common instruments:
- Bond prices (i.e., corporate bond spreads)
- Credit Default Swaps (CDS)
  - Hard arithmetic

Key assumption: event that drives the default event on the debt instrument also drives the default event in the trading book.
Corporate Bond Spreads

In a simplest case, the proceeds of investing in a risk free bond and a risky bond ($1 notional) are

\[ e^{rt} \quad \text{and} \quad e^{(r+s)t} \]

If the default probability is “p” and no recovery, then an investor should express no preference between

\[ e^{rt} \quad \text{and} \quad (1-p)e^{(r+s)t} \]
Corporate Bond Spreads

If no recovery:

\[ p = 1 - e^{-st} \]

With recovery rate \( R \):

\[ p = \frac{1 - e^{-st}}{1 - R} \]
Corporate Bond Spreads

- Default probabilities usually vary by bond maturity

<table>
<thead>
<tr>
<th>Maturity t (yrs)</th>
<th>Risk-free rate</th>
<th>Corporate Bond Spread</th>
<th>Cumulative Probability of Default</th>
<th>Annual Probability of Default (Conditional)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00%</td>
<td>1.00%</td>
<td>0.995%</td>
<td>0.995%</td>
</tr>
<tr>
<td>2</td>
<td>1.50%</td>
<td>1.25%</td>
<td>2.469%</td>
<td>1.474%</td>
</tr>
<tr>
<td>3</td>
<td>2.00%</td>
<td>1.50%</td>
<td>4.400%</td>
<td>1.931%</td>
</tr>
<tr>
<td>4</td>
<td>2.00%</td>
<td>1.50%</td>
<td>5.824%</td>
<td>1.423%</td>
</tr>
<tr>
<td>5</td>
<td>2.50%</td>
<td>1.50%</td>
<td>7.226%</td>
<td>1.402%</td>
</tr>
</tbody>
</table>

- Conditional default probabilities give information that may be overlooked in the cumulative default probabilities
Estimating Credit Losses: “…just a bit more work”

- Defaults events added into the monte carlo process allow loss events to be directly simulated.

- Assume a counterparty has a 2% chance of default within the next year.

- A uniform random variable is used to determine if the counterparty defaults:
  - Unif[0,1]
  - =RND() in Excel

- If a default occurs, a second uniform random variable determines when it happens:
  - If exposures simulated by week, time of the default given by Unif[0,52]

- The loss is simply the replacement cost of the trade less any credit offsets:
  - The average loss is the credit value adjustment (CVA) in its simplest form.
Practical Issues

- **IT issues**
  - Netting and collateral requirements, credit offsets, and ratings triggers
  - Legacy systems not designed for metrics
  - As with VaR, lots of effort required to run the model and report the results

- Potentially large computational requirements
  - However, computational resources are inexpensive

- New data requirements

- Possible regulatory interest: banks may be required to include default and funding risk when pricing derivatives
Summary

- Counterparty credit risk is a risk you already have, even if you are not measuring it

Lots to consider

- Decision to fully implement not to be taken lightly
- Exposure and funding/liquidity measures might be enough to address short term business needs
- Potential regulatory interest
Glossary

- **Current Exposure (CE)** is the larger of zero, or the market value of a transaction or portfolio of transactions within a netting set with a counterparty that would be lost upon the default of the counterparty, assuming no recovery on the value of those transactions in bankruptcy. Also called Replacement Cost.

- **Potential Future Exposure (PFE)** is a high percentile (typically 95 percent or 99 percent) of the distribution of exposures at any particular future date before the maturity date of the longest transaction in the netting set.

- **Expected Exposure (EE)** is the mean of the distribution of exposures at any particular future date before the longest maturity transaction in the netting set matures.

- **Expected Positive Exposure (EPE)** is the weighted average over time of expected exposures where the weights are the proportion that an individual expected exposure represents of the entire time interval.

- **Effective Expected Positive Exposure (Effective EPE)** is the weighted average over time of effective expected exposure over the first year, or over the time period of the longest maturity contract in the netting set where the weights are the proportion that an individual expected exposure represents of the entire time interval.

- **Peak Exposure (PE)** is a high percentile of the distribution of exposures at any particular future date before the maturity of the longest transaction in the netting set.
End Notes and References


For more information on counterparty credit risk